LIGHT-LIGHT AND HEAVY-LIGHT MESONS SPECTRA FROM NONPERTURBATIVE QCD

Alexei Nefediev

Institute of Theoretical and Experimental Physics, Moscow 117218, Russia and Centro de Física das Interacções Fundamentais (CFIF) Departamento de Física, Instituto Superior Técnico Av. Rovisco Pais, P-1049-001 Lisboa, Portugal

Abstract

Properties of light-light mesons are described by the effective Hamiltonian with spinless quarks derived from QCD. The spectrum is computed by the WKB method and shown to reproduce the celebrated linear Regge trajectories even for the lowest levels. The correct string slope of the trajectories naturally appears in the present approach as the string dynamics is taken into account properly.

Similar method is applied to heavy-light mesons and a set of corrections to the Hamiltonian is taken into account including spin-spin and Tomas spin-orbit interactions. The numerical results for the spectrum are compared with the experimental data and with the results of recent lattice calculations.

1 Introduction

One of the most successful models of confinement in QCD is the string picture which exploits the idea of the flux tube formation between the colour constituents in hadrons. The small radius of the string compared to the hadronic size makes it possible to construct quantum mechanical quark models with the interquark interaction described by either non-relativistic[1] or relativistic string (see *e.g.* [2]). The role of the string becomes especially important if light quarks are involved, so that the proper string dynamics should be taken into account together with the quarks one when studying the properties of hadrons.

2 Light-light mesons

Starting from the gauge invariant Green's function of the $q\bar{q}$ system, neglecting spins and using the Feynman-Schwinger representation for the one-particle propagators as well as the area law asymptotic for the Wilson loop one arrives at the following Lagrangian of the system

$$L(\tau)=-m_1\sqrt{\dot{x}_1^2}-m_2\sqrt{\dot{x}_2^2}-\sigma\int_0^1 d\beta\sqrt{(\dot{w}w')^2-\dot{w}^2w'^2}, \tag{1}$$

where the interaction is described by the Numbu-Goto term for the minimal string bounded by the quarks trajectories[2]. We use the straight-line anzatz for the minimal string $w_\mu(\tau,\beta) = \beta x_{1\mu} + (1-\beta) x_{2\mu}$ [2]. Introducing einbein fields μ (dynamical mass of the quark) and ν (density of the string energy) to get rid of the square roots in (1) and using the standard techniques, one finds the Hamiltonian of the system in the form (we consider equal quark masses) [2]

$$H = \frac{p_r^2 + m^2}{\mu(\tau)} + \mu(\tau) + \frac{\hat{L}^2/r^2}{\mu + 2\int_0^1 (\beta - \frac{1}{2})^2 \nu(\beta) d\beta} + \int_0^1 \frac{d\beta}{2}\left(\frac{\sigma^2 r^2}{\nu(\beta)} + \nu(\beta)\right). \quad (2)$$

Getting rid of the einbein ν by taking extremum in Hamiltonian (2) and keeping the other einbein μ as a variational parameter μ_0 one finds

$$H = \frac{p_r^2 + m^2}{\mu_0} + \mu_0 + U(\mu_0, r), \quad (3)$$

where the effective potential U has a rather complicated form; its dependence on μ_0 reflects the nonlocal string-type character of the interaction introduced in (1). Nonrelativistic expansion of (3) gives for the interquark potential

$$V(r) = U(m, r) = \sigma r - \frac{\sigma \hat{L}^2}{6m^2 r} + \dots, \quad (4)$$

where the correction to the confining potential is known as the string one [3,2].

The spectrum of the Hamiltonian (3) is found by the quasiclassical method and each eigenenergy is minimized tuning μ_0. Numerical results [4] reproduce straight-line Regge trajectories in the angular momentum l with the inverse slope $2\pi\sigma$ [4]. The difference between $2\pi\sigma$ and the overestimated value 8σ found for the Bethe-Salpeter equation with linear confinement is entirely due to the proper string dynamics missing in the latter case.

3 Heavy-light mesons

Now we apply similar approach to the heavy-light mesons spectrum. The zeroth approximation for the Hamiltonian with the Coulombic potential and the constant term added and the string correction to it read ($\kappa = \frac{4}{3}\alpha_s$):

$$H_0 = \sum_{i=1}^{2}\left(\frac{\vec{p}^2 + m_i^2}{2\mu_i} + \frac{\mu_i}{2}\right) + \sigma r - \frac{\kappa}{r} - C_0, V_{str} = -\frac{\sigma(\mu_1^2 + \mu_2^2 - \mu_1\mu_2)}{6\mu_1^2\mu_2^2}\frac{\vec{L}^2}{r}, \quad (5)$$

whereas other corrections are spin-dependent and coincide in form with the Eichten–Feinberg–Gromes results [5] up to the change $m_i \to \mu_i$ in the denominators [6] (note that for a light quark $\mu_i \sim 500 \div 800 MeV \gg m_i$):

$$V_{sd} = \frac{8\pi\kappa}{3\mu_1\mu_2}(\vec{S}_1\vec{S}_2)\,|\psi(0)|^2 - \frac{\sigma}{2r}\left(\frac{\vec{S}_1\vec{L}}{\mu_1^2} + \frac{\vec{S}_2\vec{L}}{\mu_2^2}\right) + \frac{\kappa}{r^3}\left(\frac{1}{2\mu_1} + \frac{1}{\mu_2}\right)\frac{\vec{S}_1\vec{L}}{\mu_1}$$

$$
+\frac{\kappa}{r^3}\left(\frac{1}{2\mu_2}+\frac{1}{\mu_1}\right)\frac{\vec{S}_2\vec{L}}{\mu_2}+\frac{\kappa}{\mu_1\mu_2 r^3}\left(3(\vec{S}_1\vec{n})(\vec{S}_2\vec{n})-(\vec{S}_1\vec{S}_2)\right)+\frac{\kappa^2(\vec{S}\vec{L})}{2\pi\mu^2 r^3}
$$

$$
+(2-\ln(\mu r)-\gamma_E),\quad \gamma_E=0.57. \tag{6}
$$

Numerical results for the D, D_s, B and B_s mesons spectra calculated in the the given technique with standard parameters [7,8] are in a good agreement with the experimental and lattice data.

In conclusion let us briefly discuss the situation with the $D^{*\prime}$ resonance recently claimed by DELPHI Collaboration [9]. It was reported to have the mass $2637 \pm 6MeV$ that agrees with the predictions of the quark models for the first radial excitation $2^3S_1(0^-)$ of the $q\bar{q}$ pair (our prediction for this state is $2664MeV$) but its surprisingly small width of about $15MeV$ is in a strong contradiction with the theoretical estimates [10]. Meanwhile it was observed [10] that orbitally excited states 2^- and 3^- could have such a small width. Our model predictions for these states are $2663MeV$ and $2654MeV$ correspondingly, *i.e.* they lie even lower than the radially excited one. This resolves the main objection to the identification of the $D^{*\prime}$ with orbital excitations. Indeed, in quark models 2^- and 3^- states lie at least $50-60MeV$ higher that the experimentally observed value. In our approach extra negative contribution to the masses of orbitally excited states is readily delivered by the string correction discussed above, which comes from the proper string dynamics inside meson.

Financial support of INTAS-RFFI grant IR-97-232 and RFFI grants 00-02-17836 and 00-15-96786 is gratefully acknowledged.

1. N.Isgur and S.Godfrey, Phys.Rev. **D32**, 189 (1985);
2. A.Yu.Dubin, A.B.Kaidalov, Yu.A.Simonov, Phys.Lett. **B323**, 41 (1994);
3. J.Merlin and J.Paton, J.Phys. **G11**, 439 (1985);
4. V.L.Morgunov,A.V.Nefediev,Yu.A.Simonov,Phys.Lett.**B459**,653 (1999);
5. E.Eichten and F.L.Feinberg, Phys.Rev. **D23**, 2724 (1981);
   D.Gromes, Z.Phys. **C26**, 401 (1984);
6. Yu.A.Simonov, preprint ITEP 97-89;
   J.Pantaleone, S.-H.Henry Tye, Y.J.Ng, Phys.Rev. **D33**, 777 (1986);
7. Yu.A.Simonov, Z.Phys. **C53**, 419 (1992);
8. Yu.S.Kalashnikova and A.V.Nefediev, in preparation;
9. DELPHI Collaboration, P.Abreu *et.al.*, Phys.Lett. **B426**, 231 (1998);
10. D.Melikhov and O.Pene, Phys.Lett. **B446**, 336 (1999);
    P.R.Page, Phys.Rev. **D60**, 057501 (1999)